What? I Cannot Double a Cube?

One of the all-time problems that puzzle people is why mathematicians say that a cube cannot be doubled. Well, the truth is that the problem is unsolvable only under some specific restrictions. Algebraically, this is equivalent to finding the solution for \( x \) in the equation \( x^3 = 2a^3 \) for a cube of side \( a \), but if we make \( a = 1 \), the equation is reduced to \( x^3 = 2 \). Therefore, the duplication of the cube is equivalent to finding the cubic root of 2, i.e. \( \sqrt[3]{2} \).

To double a cube, our first try is to double the base and the height of the original cube, but what we obtain is another cube 8 times the initial volume.

Geometrically, with straightedge and compass we can double a line segment and we can double square. Thus, the number 2 and \( \sqrt[3]{2} \) are constructible numbers. However, it has been proven that \( \sqrt[3]{2} \) is not constructible, no matter the efforts we make.

What are those construction restrictions? Only two:
- You must use drawing compass and straightedge only.
- The straightedge cannot be marked; that is, only pure geometric constructions are allowed, no measurements.

This problem –called the Delian problem- dates back to old Greece, originated by architectural needs.

Drawing a rectangular triangle of unit sides we obtain that the area of the square of the hypotenuse is double in size, as the area upon the unit squares.

The cube to the right has twofold volume as the cube at left.

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All compass and straightedge constructions consist of five basic constructions of points, lines, and circles:
- a line through two points,
- creating the point which is the intersection of two non-parallel lines,
- a circle through one point with centre at another point,
- creating the points of intersection of a line and a circle,
- creating the points of intersection of two circles.