WHAT? I CANNOT DOUBLE A CUBE?

One of the all-time problems that puzzle people is why mathematicians say that a **cube cannot be doubled**. Well, the truth is that the problem is unsolvable only under some specific restrictions.

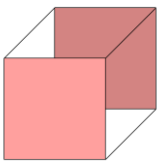
Algebraically, this is equivalent to finding the solution for x in the equation $x^3 = 2a^3$ for a cube of side a, but if we make $a = 1$, the equation is reduced to $x^3 = 2$. Therefore, the duplication of the cube is equivalent to finding the cubic root of 2, *i.e.* $\sqrt[3]{2}$.

What are those construction restrictions? Only two:

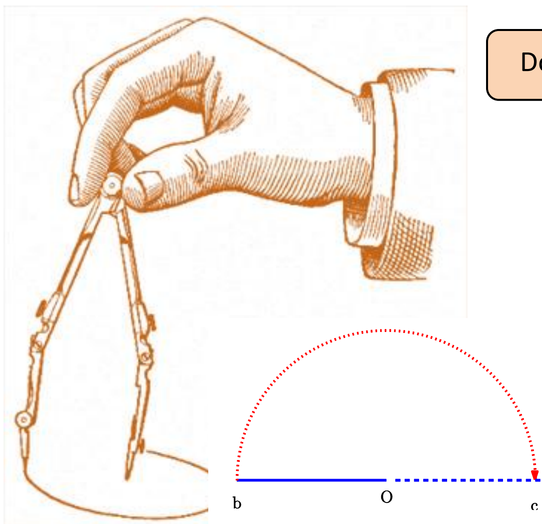
- You must use drawing **compass and straightedge only**.
- The **straightedge cannot be marked**; that is, only pure geometric constructions are allowed, no measurements.

This problem –called **the Delian problem**- dates back to old Greece, originated by architectural needs.

Drawing a rectangular triangle of unit sides we obtain that the area of the square of the hypotenuse is double in size as the area upon the unit squares.

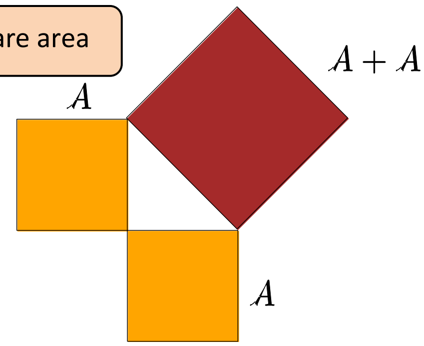


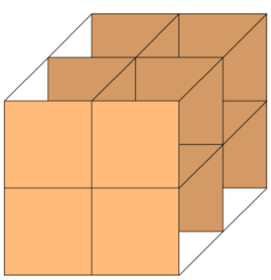
To double a cube, our first try is to double the base and the height of the original cube, but what we obtain is another cube 8 times the initial volume.



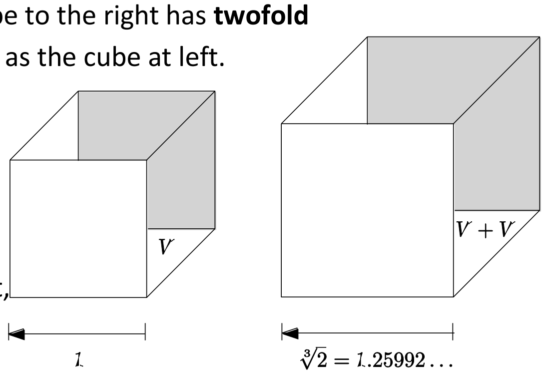
Doubling a line segment

Doubling a square area





The cube to the right has **twofold** volume as the cube at left.



Doubling a cube

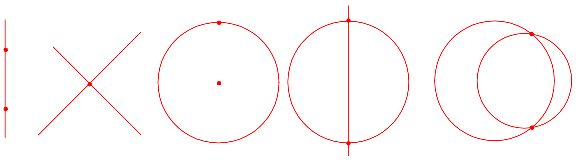
Geometrically, with straightedge and compass we can double a line segment, and we can double square. Thus, the number 2 and $\sqrt{2}$ are **constructible numbers**. However, it has been proven that $\sqrt[3]{2}$ **is not constructible**, no matter the efforts we make.

Pierre Laurent Wantzel (1814-1848) proved that many geometrical problems are **unsolvable** if using only unruled compass and straightedge.

$$\sqrt[3]{2} =$$

1.2599210498948731
647672106072782 …

$$\sqrt[3]{2} \times \sqrt[3]{2} \times \sqrt[3]{2} = 2$$



All compass and straightedge constructions consist of **five basic constructions** of points, lines, and circles:

- a line through two points, • creating the point which is the intersection of two non-parallel lines, • a circle through one point with centre at another point, • creating the points of intersection of a line and a circle. • creating the points of intersection of two circles.

Reference: Allman, G. (1877). *Greek Geometry from Thales to Euclid*. Dublin University Press.